NUCLEAR-COUPLED THERMAL-HYDRAULIC STABILITY AND BIFURCATION ANALYSES OF FORCED- AND NATURAL-CIRCULATION BWRS

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ABSTRACT

We report our recent efforts in modeling nuclear-coupled thermal hydraulic instabilities in forced and natural circulation boiling water reactors characterized by single and two-phase flows. While density-wave type oscillations are captured with thermal hydraulic models with constant properties, simulation of flashing requires the water saturation enthalpy to be treated as a function of local pressure. Hence, separate thermal hydraulic models are developed and used for the two different kinds of instabilities. Models also include fuel rod heat conduction and modal neutron kinetics. Results of steady-state, stability and bifurcation analyses, as well as numerical integration are reported. Preliminary results of numerical integration to simulate intermittent oscillations in natural circulation heated loops with two-phase flow are also reported. Results show that these models can capture in-phase and out-of-phase oscillations as well as show sub- and supercritical bifurcations as stability boundaries are crossed.

KEYWORDS: natural circulation, forced, BWR, stability

1. INTRODUCTION

Current design of boiling water reactors (BWRs) relies on recirculation pumps to remove fission heat from cores. New designs of BWRs relying on natural circulation to extract core heat, such as SBWR and ESBWR, have been completed, and they are promising candidates to replace the current forced-circulation BWRs. While forced circulation BWRs faced nuclear-coupled thermal hydraulic stability problems [1] in an island in the power-flow plane—under high power and low flow conditions—similar concerns exist for natural circulation BWRs during startup conditions. In addition to the complicated nuclear-coupled density-wave oscillations, natural circulation systems with their long risers must also address the issue of flashing and associated nuclear-coupled thermal hydraulic stability. Flashing here means boiling in natural circulation loop caused not by heating but by local pressure drop usually in the long riser [2]. In this case, small perturbation of flow rate can be dramatically intensified by vaporization due to pressure changes.

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Large scale and high fidelity commercial codes have been used to study nuclear-coupled density wave oscillation (DWO) phenomena in BWRs for two decades. Takigawa et al. [3] simulated the out-of-phase oscillations observed in Caorso-I with a three-dimensional code, TOSDYN. Araya et al. [4] studied the instability event that occurred in Lasalle-2 using RETRAN. Hennig [5] carried out analyses of instability events and tests in Ringhals, KKL, and Forsmark employing three-dimensional BWR transient simulation code, RAMONA. Additional references can be found in other papers presented in this session. Although such detailed simulations and analyses are very helpful in understanding specific events, they are cumbersome, time-consuming, and of limited value in determining stability boundaries in large parameter spaces, and in predicting parametric trends. Moreover, few of the codes take into account dependence of water saturation enthalpy on local pressure, which is key to simulating the flashing phenomenon. Therefore, they are not capable of simulating dynamics of natural circulation BWRs under low pressure conditions. For these reasons, custom-developed reduced order models (ROMs) are required.

Focus of our recent work has been on the development of ROMs for forced and natural circulation BWRs. We have used these ROMs to study steady-state, and stability characteristics of BWRs. We first [6] used a reduced-order model, originally developed by Karve et al. [7], to study forced-circulation BWRs. The model is modified to take in-homogeneity of core loading into considerations. Stability maps and properties of Poincare-Andronov-Hopf bifurcations are calculated and analyzed. Next, we developed [8] a new model to study dynamics of natural circulation BWRs. Water saturation enthalpy in this model depends on local pressure, allowing analysis of the flashing phenomenon.

After giving a brief review of past work, we present results of numerical integration of the governing equations to simulate long-period intermittent oscillations in natural circulation heated systems at low pressures.

2. REDUCED ORDER MODELS

2.1 Neutron kinetics and Heat Conduction Models

Reduced order models consist of sub-models for neutron kinetics, fuel heat conduction, and the single- and two-phase flow in the heated channel or in the loop. Neutron kinetics and fuel rod heat conduction models do not differ for forced and natural circulation systems. Neutron kinetics for both is based on the modal expansion approach (ω-mode) [7]. The final form of the modal kinetics equations are

\[
\frac{dn_k(t)}{dt} = \omega_{ik} n_k(t) + (\omega_{ik} - \omega_{0k}) u_k(t) + \sum_{m=0}^{1} \rho_{km} \Lambda_k n_m(t)
\]

\[
\frac{du_k(t)}{dt} = \omega_{ik} u_k(t) + \sum_{m=0}^{1} \rho_{km} \Lambda_k n_m(t)
\]

where \(n_0\) and \(n_1\) are non-dimensional neutron densities in the fundamental and first azimuthal modes, respectively, \(u_0\) and \(u_1\) are non-dimensional precursor densities, and \(\Lambda_{ik}\) is the effective neutron generation time. The neutron kinetics equations are coupled with fuel rod heat conduction and channel thermal-hydraulics equations through reactivity feedbacks.
\[
\rho_{km}(t) = \sum_{l=0}^{i} \rho_{ext,km} - c_{\alpha,km}(\alpha_l(t) - \alpha_{0,l}) - c_{D,km}(T_{avg,l}(t) - T_{avg,0,l})
\]  
(2)

where \( \rho_{ext,km} \) is the control-rod induced reactivity, and \( \alpha_l(t) \) and \( T_{avg,l}(t) \) are average void fraction and average fuel temperature of channel \( l \), respectively. To take into account the impact of in-homogeneity in core loadings, an amplification factor \( F \) is introduced to modify reactivity feedbacks from azimuthal mode

\[
c_{\alpha,lm,nu} = F \cdot c_{\alpha,lm,ut}
\]  
(3)

\[
c_{D,lm,nu} = F \cdot c_{D,lm,ut}
\]  
(4)

where, \( c_{\alpha,lm,nu} \) and \( c_{D,lm,nu} \) are feedback coefficients of a non-uniform core; and \( c_{\alpha,lm,ut} \) and \( c_{D,lm,ut} \) are feedback coefficients of a uniform core. If the in-homogeneity is small, \( F \) is close to 1, and reactivity feedback from the azimuthal mode is small. On the other hand, if the in-homogeneity is large, value of \( F \) is also large indicating a strong feedback from the azimuthal mode.

Heat conduction model is developed using the variational method [7]. Due to large temperature gradient, a two-region piecewise quadratic temperature distribution is assumed in the fuel rod. Starting from the time-dependent heat conduction equations in cylindrical coordinates, variational principle is used to derive the set of ODEs for the expansion parameters of the quadratic profile [7].

The heat conduction and channel thermal-hydraulic sub-models are coupled with the neutronics sub-model by the heat generation rate

\[
q_l''(t) = c_q(n_0(t) + \xi_l n_1(t))
\]  
(5)

where \( c_q \) is a constant that converts the dimensionless neutron density to power density. Volume factor \( \xi_l \) denotes the contribution of channel \( l \) to the total power.

2.2. Single- and Two-Phase Thermal Hydraulics Model

The thermal hydraulics model for the forced circulation BWRs was developed by Karve et al and used earlier by Zhou and Rizwan-uddin [6] to study bifurcation characteristics of the forced systems. It will not be presented here in detail. Briefly, in developing the heated channel thermal hydraulics model, quadratic spatial variations are assumed for water enthalpy in the single-phase region, and for steam quality in the two-phase part. Homogeneous two-phase flow model is used to model the two-phase flow. Ordinary differential equations (ODEs) for the expansion coefficients of the quadratic spatial variation are derived from the energy equations in the single phase and two-phase regions using the weighted-residual approach. The momentum equations are integrated over the entire length of the flow channel to obtain the ODE for the inlet velocity.
2.2.1 Thermal Hydraulics of the Natural Circulation System

The system is shown in Figure 1. It is comprised of a heated channel (or core), an unheated riser above the channel, and a downcomer. A system pressure is imposed by setting the pressure at outlet of the riser, $P_{exit}$, to a fixed value. $q'$ is the heat flux per unit length into the channel, $k_{c,in}$, $k_{c,out}$, $k_{r,out}$, and $k_{dc,in}$ are local pressure loss coefficients.

![Figure 1. Schematic diagram of a natural circulation system.](image)

The thermal-hydraulic model is based on the following assumptions:

- Saturation enthalpy varies linearly with (local) pressure. Other thermal properties of water and steam are constant.
- Subcooled boiling and carry-under effects are ignored.
- Two phase flow can be modeled by the homogeneous equilibrium model (HEM).

Although these assumptions can be relaxed in favor of quadratic pressure dependence and non-equilibrium two-phase flow models, it will greatly increase the complexity and decrease computational efficiency of the model. Ignoring subcooled boiling may be a more important limitation of this model.

The channel and riser are discretized into nodes. Figure 2 shows the moving boundary scheme, where node boundary between the last single-phase node and the first two-phase node moves together with the boiling boundary, thus avoiding zero-length node. Three cases, based on the location of the boiling boundary, must be considered. These are shown in Figures 2(a), (b) and (c). For case 1, water enthalpy at the core exit is less than the local saturation enthalpy, and flashing occurs in the riser. For case 2, because of relatively large heat input, water starts boiling in the core. Case 3 is a degenerate case, in which water enthalpy is less than the saturation value at the core exit, but greater at the entrance of the riser due to local pressure loss.
2.2.2. Model Development

Non-dimensional forms of continuity, energy and momentum partial differential equations are used to derive the dynamical system. These are equations (6-9),

\[
\frac{\partial\tilde{x}(z,t)}{\partial t} = (N_p N_r + x(z,t)) \frac{\partial\tilde{v}_m(z,t)}{\partial z} - \tilde{v}_m(z,t) \frac{\partial\tilde{x}(z,t)}{\partial z}
\]

\[
\frac{\partial h_m(z,t)}{\partial t} = -\left( \frac{N_{pc}^{-1}(t)}{N_{flash}^{-1}} \left(1 + \frac{x(z,t)}{N_p N_r} \right) + \tilde{v}_m(z,t) \frac{\partial h_m(z,t)}{\partial z} \right)
\]

\[
\frac{\partial h_f(z,t)}{\partial t} = -\left( \frac{\tilde{v}_m(z,t) \left(h_e - h_f(z,t) \right) \left(N_p N_r + x(z,t) \right)}{N_{pc}^{-1}(t) \left(N_p N_r + x(z,t) \right)} + \frac{\partial h_f(z,t)}{\partial z} \right)
\]
\[
\frac{\partial v_m(z,t)}{\partial t} = \Delta P_{\text{drv}} \frac{N_\rho N_e + x(z,t) \partial h_f(z,t)}{N_\rho N_e} \frac{\partial h_f(z,t)}{\partial z} - \\
\left( v_m(z,t) \frac{\partial v_m(z,t)}{\partial z} + N_f \sum_{m=1}^{M_m} k_m v_m(z,t)^2 \delta(z - z_m) \right)
\]

where, \( x \) is steam quality, \( h_m \) is flow enthalpy, \( h_f \) is water saturation enthalpy, and \( v_m \) is flow velocity. Note that pressure dependence of water saturation enthalpy leads to modified forms of energy and momentum equations. A weighted residual approach is employed to reduce the PDEs to a set of ODEs. In each node, spatially linear trial functions with time-dependent parameters for saturated enthalpy, mixture enthalpy, steam quality, and mixture velocity are introduced,

\[ h_{m,i}(z,t) = h_{m,i-1}(z_{i-1}(t), t) + a_{m,i}(t)(z - z_{i-1}(t)) \]  \hspace{1cm} (10)

\[ h_{f,i}(z,t) = h_{f,i-1}(z_{i-1}(t), t) + a_{f,i}(t)(z - z_{i-1}(t)) \]  \hspace{1cm} (11)

\[ x_{i}(z,t) = x_{i-1}(z_{i-1}(t), t) + N_\rho N_e s_{i}(t)(z - z_{i-1}(t)) \]  \hspace{1cm} (12)

\[ v_{m,i}(z,t) = v_{m,i-1}(z_{i-1}(t), t) + v_{i}(t)(z - z_{i-1}(t)) \]  \hspace{1cm} (13)

ODEs for the time-dependent expansion parameters \( a_m(t), a_f(t), s(t) \) and \( v(t) \) are derived by integrating the governing PDEs along the node. Their generic forms are given by

\[ \frac{da_m(t)}{dt} = \frac{A_{m,i}(a_m, a_f, v, s, \ldots)}{\Delta z} + A_{m,2}(a_m, a_f, v, s, \ldots) + \ldots \]  \hspace{1cm} (14)

\[ \frac{da_f(t)}{dt} = \frac{A_{f,i}(a_m, a_f, v, s, \ldots)}{\Delta z} + A_{f,2}(a_m, a_f, v, s, \ldots) + \ldots \]  \hspace{1cm} (15)

\[ \frac{ds(t)}{dt} = \frac{S_{i}(a_m, a_f, v, s, \ldots)}{\Delta z} + S_{2}(a_m, a_f, v, s, \ldots) + \ldots \]  \hspace{1cm} (16)

\[ \frac{dv(t)}{dt} = \frac{V_{i}(a_m, a_f, v, s, \ldots)}{\Delta z} + V_{2}(a_m, a_f, v, s, \ldots) + \ldots \]  \hspace{1cm} (17)
where $A_m, A_f, S$ and $V$ are nonlinear functions of $a_m(t), a_f(t), s(t)$ and $v(t)$. Their detailed forms will be given in [10]. In discretization schemes where all node lengths are fixed ($\Delta z$) except for the last single-phase and the first two-phase nodes—these nodes are divided by a moving boiling boundary, and their lengths should add up to $\Delta z$—the length of the last single-phase node or the first two-phase node can be very small. RHSs of equations (14-17) are written in this fashion to explicitly bring out the difficulty that arises in fixed boundary discretization schemes as the node length $\Delta z$ approaches zero. A lower limit on node size and a stable numerical integration scheme are used to avoid numerical difficulties. Boiling boundary and its derivative are evaluated by applying the fixed pressure condition at the riser exit. The set of ODEs are solved for steady-state, stability and bifurcation analyses.

3. RESULTS: STABILITY AND BIFURCATION ANALYSES

3.1 Forced Circulation BWRs

We first give a brief review of results obtained for the forced circulation system. Eigenvalues of the Jacobian of the forced-circulation BWR model shows that pairs of complex conjugate eigenvalues with the two largest real parts have the biggest effect on the stability of the system and on the kind of oscillations that are likely to result as the stability boundary (SB) is crossed. Ratio of the element corresponding to $n_0$ (fundamental mode expansion parameter) to that for $n_1$ (first azimuthal mode expansion parameter) in the corresponding eigenvector is much larger than 1 for one of these eigenvalues, and much smaller than 1 for the other. Either one of these two pairs of eigenvalues—depending upon the parameter values—may be the dominant one. To avoid confusion, the pair of eigenvalues $e_1$ for which magnitude of the element of $n_0$ in the corresponding eigenvector is much larger than that of element of $n_1$, will be called the fundamental mode eigenvalues ($a_f + i w_f$), and the other pair $e_2$ will be called the first azimuthal mode eigenvalues ($a_a + i w_a$). It is clear that if SB is determined by $e_1$, the amplitude of $n_0$ in the resulting oscillations will be much larger than that of $n_1$. The oscillations will be predominantly in-phase. However, if SB is determined by $e_2$, the amplitude of $n_1$ in the resulting oscillations will be much larger than that of $n_0$. The oscillations will be predominantly out-of-phase.

Figure 3(a) shows the SB in $N_{sub} - \Delta P_{ext}$ space for $F = 5.0$. The SB (solid line) in Figure 3(a), on which the real part of the rightmost eigenvalues is zero, is comprised of two curves that meet at the point T ($N_{sub} = 1.177$, $\Delta P_{ext} = 7.618$). Above this point, the eigenvalues with the largest real part along the SB are the fundamental mode eigenvalues, that is, $a_f = 0$ and $a_a < 0$. However, for $N_{sub} < 1.177$ (below the point T), the eigenvalues with the largest real part along the SB are the azimuthal mode eigenvalues ($a_a = 0$ and $a_f < 0$). The dotted line in Figure 3(a) denotes the boundary along which the real part of the second rightmost pair of complex conjugate eigenvalue is zero. (This means that the rightmost pair of complex conjugate eigenvalue along the dotted line has positive real part). The dotted curve is also comprised of two branches. Along the branch above the point T, the real parts of the azimuthal mode eigenvalues...
are zero ($a_a = 0$ and $a_f > 0$). However, along the lower branch, the real parts of the fundamental mode eigenvalues are zero ($a_f = 0$ and $a_a > 0$). Hence, the parameter space is divided into four parts, denoted by letters A, B, C and D. Region C and A are respectively the stable and unstable regions. Regions B and D are also unstable. However, in region B, the fundamental mode eigenvalue has positive real part while first azimuthal mode eigenvalue has negative real part, and therefore resulting oscillations will be predominantly in-phase. Similarly, in region D, the first azimuthal mode eigenvalue has positive real part while fundamental mode eigenvalue has negative real part, and therefore resulting oscillations will be predominantly out-of-phase. is unstable while the first azimuthal mode (out-of-phase) is stable. These conclusions are confirmed by numerical integration of the nonlinear dynamical system [6].

Results of the bifurcation analyses for $F = 5$ are presented in Figure 3(b) in the form of 5% oscillation amplitude curve in $N_{sub} - (\Delta P_{ext} - \Delta P_{ext,critical})$ space. The solid line corresponds to the SB (solid line) in Figure 3(a), while the dotted curve corresponds to the dotted curve in Figure 3(a) (i.e., to the second rightmost eigenvalue crossing the imaginary axis). [The bifurcation is subcritical for $N_{sub} > 1.874$.] In Figure 3(b), a jump in the 5% oscillation curve (solid line) occurs at the value of $N_{sub}$ corresponding to point T. The discontinuity exists because of the degeneracy at this value of $N_{sub}$, where both the fundamental and first azimuthal mode eigenvalues have zero real parts. The 5% oscillation curve for $N_{sub} > 1.177$ is determined using the fundamental mode eigenvalues, while for $N_{sub} < 1.177$ they must be determined using the first azimuthal mode eigenvalues.

### 3.2 Natural Circulation BWRs
As shown above for forced circulation BWR system, the two rightmost pairs of eigenvalues can be associated with the in-phase and out-of-phase oscillations. Figures 4 and 5 show the two boundaries in $\Delta T_{\text{inlet}} - \rho_{\text{ext}}$ space for system pressures of 7.0 MPa and 0.4 MPa, respectively, for the natural circulation system. The stability boundaries (solid lines) correspond to the operating points on which the largest real part of all eigenvalues is zero. The second set of boundaries (dotted lines), however, are boundaries on which the second largest real part is zero. For low system pressure (Figure 5) the two boundaries do not intersect, and hence the entire SB is composed of the boundary associated with one pair of eigenvalue—the fundamental mode eigenvalue, in this case the one with squares. The second boundary is composed of that associated with the fundamental mode eigenvalue, in this case the one with triangles. However, Figure 4, for $P_{\text{exit}} = 7.0$ MPa, shows that the two boundaries intersect at point X ($\rho_{\text{ext}} = -0.0335$, $\Delta T_{\text{inlet}} = 8.65$ K). Hence, the SB is composed of a segment of the boundary X-B-C (with squares) associated with the fundamental mode eigenvalue and another segment A-X (with triangles) associated with the azimuthal mode eigenvalue. Therefore, a characteristic change in the nature of oscillation along the SB (solid line) must be expected due to eigenvalue crossing at point X (Figure 4). To the left of point X, segment A-X is the SB due to out-of-phase mode eigenvalue. To the right of point X, segment X-B-C is the SB due to the in-phase mode eigenvalue.

**Figure 4.** SB and boundary along which the real part of the second largest eigenvalue is zero ($P_{\text{exit}} = 7.0$ MPa).
Figure 5. SB and the boundary along which the real part of the second largest eigenvalue is zero ($P_{\text{exit}} = 0.4$ MPa).

Parameter space in Figure 4 is divided by these boundaries of fundamental and first azimuthal mode eigenvalues into four regions. The system in region I is stable. Small perturbations will cause decreasing amplitude oscillations. The system in region II is unstable. Here the fundamental mode eigenvalue has positive real part while first azimuthal mode eigenvalue has negative real part. The oscillations in this region will therefore be dominated by the fundamental mode. In region III, the first azimuthal mode eigenvalue has positive real part while fundamental mode eigenvalue has negative real part. The oscillations in this region are therefore, predominantly out-of-phase. In the last region (IV), both eigenvalues have positive real part and therefore oscillations will be a combination of in-phase and out-of-phase modes.

For low system pressure, instead of four regions, three regions of the parameter space are identified. The system is stable in region I. Region II is unstable and oscillations are expected to be predominantly in-phase. Both modes are unstable in region III (Figure 5).

Figures 6 and 7 show 7.5% oscillation curves for different segments of the SBs of Figures 4 and 5 in $\Delta T_{\text{inlet}} - (\rho_{\text{ext}} - \rho_{\text{ext, critical}})$ parameter space. Here $\rho_{\text{ext, critical}}$ is the reactivity on the SB. The amplitude of the limit cycle along these curves is about 7.5% of the magnitude of eigenvector elements. In Figure 6(a), for the out-of-phase segment A-X, the oscillation curve is in the unstable region, indicating a supercritical PAH-B along the SB. From the out-of-phase segment (A-X) of the SB to in-phase segment X-B, oscillation curve has a jump from the unstable side to the stable side. For $\Delta T_{\text{inlet}} < 28.91$ K, the oscillation curve of segment X-B is in the stable region, and type of PAH-B is subcritical. For $28.91 < \Delta T_{\text{inlet}} < 31.42$ K (point B), the oscillation curve returns to the unstable region, and consequently type of PAH-B along this segment of the SB is supercritical. For segment B-C, Figure 6(c) shows that oscillation curve along the B-C branch is always in the unstable region. Type of PAH-B along this segment is hence supercritical. Similar back-and-forth transitions from sub- to supercritical bifurcations are observed for the low system pressure case. In Figure 7(a), the type of bifurcation is supercritical along the D-E branch of the
SB. Along the E-F branch of the SB (Figure 7(b)), when $\Delta T_{inlet} > 2.762$ K, the bifurcation is supercritical; while for $\Delta T_{inlet} < 2.762$ K, it is subcritical.

![Figure 6. 7.5% oscillation curve along three segments of the SB in Figure 4.](image_url)

![Figure 7. 7.5% oscillation curve along two segments of the SB in Figure 5.](image_url)

For the out-of-phase segment A-X (Figure 6(a)) and low $\Delta T_{inlet}$ region of the in-phase segment B-C (Figure 6(c)), deviations of $\rho_{ext}$ from its critical value required for the 7.5% amplitude, are much smaller than those for the segment X-B and high $\Delta T_{inlet}$ region of the segment B-C, indicating that much larger amplitude oscillations (in-phase or out-of-phase) may result for the same deviation along the SB in the low subcooling region than those along the SB in the high subcooling region. Large amplitude oscillations at lower inlet subcooling are due to longer two-phase regions in the channel and riser. Similar trend can be found in the low system pressure case (Figure 7).

The change in characteristic of bifurcation (supercritical or subcritical) and change in characteristic of dominant mode of oscillations (in-phase or out-of-phase) along the SB, thus, reveal complexity of the dynamic behavior of the coupled natural circulation BWR system.

### 4. NUMERICAL SIMULATIONS

Numerical simulations were carried out to gain further insight into the dynamics of the coupled system as well as to examine the results obtained using BIFDD for the forced as well as natural
circulation BWR models. Due to space limitation, we here only present the results of numerical integration of the natural circulation system. Eight points on the stability maps of high and low pressure systems are chosen for numerical simulations. These points are shown in Figures 8 (a) and (b). Table I lists operating parameters as well as characteristics of bifurcation and nature of oscillations (in-phase or out-of-phase) observed in simulations. These results agree with the results obtained using the semi-analytical bifurcation analysis carried out using the BIFDD code.

In addition to the high frequency density-wave type oscillations, two-phase, natural circulation heated channel loops are also known to experience low frequency intermittent oscillations [14-16]. As a first step toward numerical integration analysis of long-period intermittent oscillations in natural circulation BWRs at low pressures, we have first studied the thermal hydraulic system without neutronics. Simulation of loop dynamics under these conditions is a challenging task. Period of oscillation can be in the range of 100s [14] [15]. Simulation is made even more challenging by the boiling boundary dynamics. In case of intermittent oscillations, the boiling boundary may reach the top of the riser (subcooled liquid in both, the core and the riser) and may stay there for tens of seconds. As the flow rate and core inlet temperature change, boiling starts at the top of the riser and “spreads” downward. After reaching a minimum value the boiling boundary shoots back up to the core exit. The cycle then repeats. [Manera et al [14] and others have also reported cases in which boiling actually starts in the middle of the riser, and spreads both upward and downward as the two-phase region expands. This case happens when, during the oscillations, while both core and riser are filled with subcooled liquid, a wave of high temperature liquid travels upward and the temperature at the peak of this wave becomes equal to the saturation temperature somewhere in the riser. In this case, while fluid above and below is subcooled, the fluid at the peak temperature starts boiling. Our model currently cannot simulate this scenario.] Figure 9 shows the inlet velocity and boiling boundary variation during intermittent oscillations in a natural circulation two-phase flow loop. The oscillation period is close to 50 s.

![Figure 8](image)

*Figure 8. Stability boundaries and operating points for numerical simulations (a) $P_{exit} = 7.0$ MPa (same as the SB in Fig. 4); (b) $P_{exit} = 0.4$ MPa (same as the SB in Fig. 5).*
Table I. Operating points for numerical simulations

<table>
<thead>
<tr>
<th></th>
<th>$P_{exit}$ (MPa)</th>
<th>$\rho_{ext}$</th>
<th>$\Delta T_{inlet}$ (K)</th>
<th>Characteristics of oscillations</th>
<th>Characteristics of PAH-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point $a$</td>
<td>7.0</td>
<td>-0.03413</td>
<td>6.0</td>
<td>Out-of-phase</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Point $b$</td>
<td>7.0</td>
<td>-0.03400</td>
<td>6.0</td>
<td>Out-of-phase</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Point $c$</td>
<td>7.0</td>
<td>-0.02781</td>
<td>20.0</td>
<td>In-phase</td>
<td>Subcritical</td>
</tr>
<tr>
<td>Point $d$</td>
<td>7.0</td>
<td>-0.00840</td>
<td>20.0</td>
<td>In-phase</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Point $e$</td>
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<td>-0.00823</td>
<td>20.0</td>
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<td>Supercritical</td>
</tr>
<tr>
<td>Point $f$</td>
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<td>0.01237</td>
<td>2.0</td>
<td>In-phase</td>
<td>Subcritical</td>
</tr>
<tr>
<td>Point $g$</td>
<td>0.4</td>
<td>-0.02570</td>
<td>3.8</td>
<td>In-phase</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Point $h$</td>
<td>0.4</td>
<td>-0.02540</td>
<td>3.8</td>
<td>In-phase</td>
<td>Supercritical</td>
</tr>
</tbody>
</table>

Fig. 9. Intermittent oscillations in natural circulation loop (no neutronics). (a) inlet velocity, (b) boiling boundary.

SUMMARY

We have reported recent developments in stability analyses of forced and natural circulation BWRs. These include stability, bifurcation and numerical integration of reduced order models. These studies can be relevant and easily extended to carry out stability analysis of super critical water reactor.

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