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BWR stability and bifurcation analysis using reduced order models and system codes: Identification of a subcritical Hopf bifurcation using RAMONA

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Abstract

The system code RAMONA, as well as a recently developed BWR reduced order model (ROM), are employed for the stability analysis of a specific operational point of the Leibstadt nuclear power plant. This has been done in order to assess the ROM's applicability and limitations in a quantitative manner.

In the context of a detailed local bifurcation analysis carried out using RAMONA in the neighbourhood of the chosen Leibstadt operational point, a bridge is built between the ROM and the system code. This has been achieved through interpreting RAMONA solutions on the basis of the physical mechanisms identified in the course of applying the ROM. This leads, for the first time, to the identification of a subcritical Poincaré–Andronov–Hopf (PAH) bifurcation using a system code. As a consequence, the possibility of the so-called correspondence hypothesis is suggested to underline the relationship between a stable (unstable) limit cycle solution and the occurrence of a supercritical (subcritical) PAH bifurcation in the modeling of boiling water reactor stability behaviour.

1. Introduction

An extended BWR reduced order model[2] (ROM) was developed and applied, by Dokhane et al. (2007), to simulate global and regional oscillations for typical values of BWR operating and design parameters. Thereby, stability and bifurcation analyses were performed using the bifurcation analysis code BIFDD (Hassard, 1987). Stability boundaries and the nature of PAH bifurcation were determined and presented in a suitable two-dimensional parameter-state space. From an in-depth investigation of the elements of the eigenvectors of the system, it was shown that analysing the properties of the elements of the eigenvectors corresponding to the pairs of complex eigenvalues with the largest and second largest real parts gives detailed information on the type of oscillation mode, i.e. in-phase or out-of-phase, without solving the corresponding set of ODEs.

The principal objective of the current work is to compare the ROM results with those obtained using a large system code, viz. RAMONA (Wulf et al., 1984), for a common BWR operational point (OP). The direct comparisons between the PSI ROM and the system code, with its much higher degree of NPP modeling detail, should allow important conclusions to be drawn regarding the ROM's applicability and limitations.

Various previous studies using reduced order models (Muñoz-Cobo and Verdú, 1991; Tsuji et al., 1993; Karve et al., 1997; van Bragt et al., 1999, 2000; Zboray et al.,

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[2] Called later the PSI ROM.

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1999), including the analysis reported by Dokhane et al. (2007) with the PSI ROM, show that both sub- and supercritical PAH bifurcations can be expected during the loss of system stability in a BWR. These in fact being the only bifurcation types encountered in such analyses while crossing the stability boundary.[3]

In BWR stability analysis using system codes, stable limit cycle oscillations have been observed and reported (Lefvert, 1996; Hennig, 1999; Miró et al., 2000), as has also been the case for certain BWR stability tests performed at NPPs such as Leibstadt and Ringhals (Blomstrand, 1992; Johansson, 1994). To the authors' knowledge, unstable limit cycles, however, have not been observed or reported in numerical studies using system codes. Clearly, unstable limit cycles, in a numerical analysis, cannot be achieved by marching forward in time. Their existence can only be inferred by studying the system response to different amplitude perturbations.

Within the framework of reduced order model semi-analytical bifurcation analysis, results clearly distinguish stable fixed points with an unstable limit cycle around them from unstable fixed points with stable limit cycle around them. With this information in hand, confirmation of the results using numerical integration is straightforward. Thus, for an unstable fixed point, whatever the initially induced perturbation amplitude, oscillations always grow in amplitude. On the other hand, for unstable limit cycles, the initial perturbation amplitude plays an important role in determining the behaviour of the system, viz. for small perturbation amplitudes the oscillations decay to the fixed point that exists inside the unstable limit cycle, whereas for a large enough perturbation amplitude the oscillations grow in amplitude. Such an exercise using a system code is very time consuming. However, if semi-analytical bifurcation analysis results of an "approximate" ROM are available, they can be judiciously used to help "zoom-in" the area of interest (subcritical bifurcation) for a more detailed analysis using the system code.

In this paper, in Section 2, analysis using the PSI ROM is carried out for a particular operational point (OP) of the Leibstadt nuclear power plant (NPP) in Switzerland, and stability and bifurcation characteristics such as stability boundary, nature of bifurcation and type of oscillation mode are determined. In Section 3, the same Leibstadt OP, as that analysed using the ROM, is analysed using the system code RAMONA. The primary focus in Section 3 is to address the RAMONA solutions in the light of the physical mechanisms identified in the course of applying the ROM. Thus, a detailed numerical bifurcation analysis is carried out with the system code by carefully examining the solutions obtained at several different OPs in the neighbourhood of the reference Leibstadt OP. A qualitative link is thereby established between the ROM and RAMONA, resulting in the identification of a subcritical PAH bifurcation. To the authors' knowledge, this is the first demonstration of a subcritical bifurcation using a large scale BWR system code.

A summarized assessment of the PSI ROM against the results obtained with the system code is given in Section 4, while final conclusions are drawn in Section 5.

2. Stability and bifurcation analysis of the kklc7_rec4 OP using the PSI reduced order model

In this section, as indicated earlier, the stability behaviour of a specific OP of the Leibstadt NPP is analysed using the PSI ROM (Dokhane et al., 2007). This OP corresponds to the so-called cycle 7 record 4 (kklc7_rec4 OP), with 60.5% power and 36.7% mass flow rate. The OP is located in the plan's exclusion area,[4] in the power–flow plane, and is an OP for which a stability measurement was carried out during cycle 7 reactor start-up in September 1990 showing growing out-of-phase oscillation amplitudes.

For the ROM modeling, the operating and design parameters for this OP have been evaluated suitably. For instance, the single and two-phase friction factors, the two-phase multiplier, the fuel heat capacity, the thermal fuel conductivity and the gap conductance have all been calculated using the correlations used in RAMONA. The inlet and exit loss coefficients have been adjusted to include the spacer pressure losses in the channel at the inlet and exit. The complete set of design and operating parameters for kklc7_rec4 OP is given in Dokhane (2004).

Fig. 1 gives the stability boundary as predicted by the reduced order model for kklc7_rec4 OP in the N_{sub}–DP_{ext} plane. At the kklc7_rec4 OP:

- The subcooling enthalpy is 125×10^3 J kg^{-1}, which corresponds to a subcooling temperature of 23.4 K, and a subcooling number N_{sub} of 1.55 (all the dimensionless quantities are defined in Dokhane (2004)).
- The total pressure drop across the core is 0.4497×10^5 N/m^2. This corresponds to a dimensionless total pressure drop across the core DP_{ext} of 8.57.

The estimation of the void distribution parameter and drift velocity values at this OP is based on the following justifications:

(i) The thermal-hydraulic model validation, mentioned in Dokhane et al. (2007), was carried out mainly against the Saha et al. (1976) experimental data. It has been shown that, for the Set I data (corresponding to an inlet velocity of 0.98 m/s), a value of the void distribution parameter of 1.03 allows the employed model to predict a SB which best fits the

[3] Deep inside the unstable region, a cascade of period-doubling bifurcations may exist as reported, for instance, by van Bragt et al. (1999).

[4] A conservatively defined region in the power–flow map where the reactor is not allowed to operate during normal operating conditions.

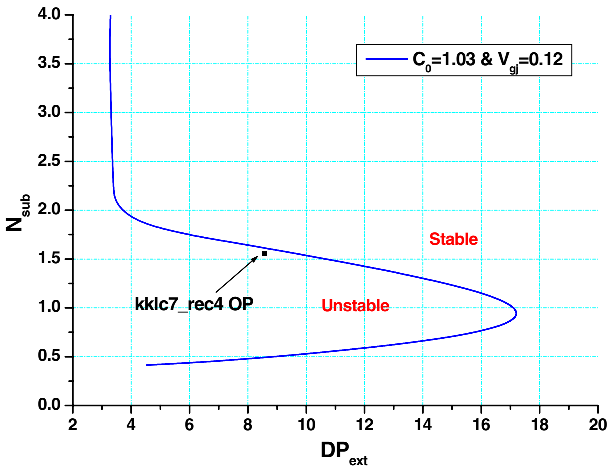

Fig. 1. Stability boundary in N_{sub}–DP_{ext} plane for operating and design parameters corresponding to the kklc7_rec4 operational point (ROM).

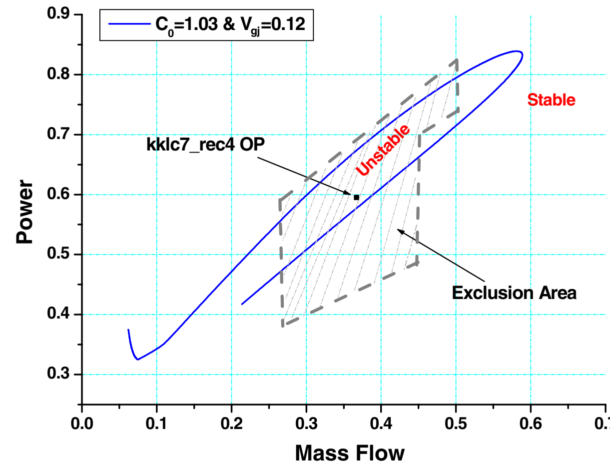

Fig. 2. Stability boundary in the power–flow map for operating and design parameters corresponding to the kklc7_rec4 operational point (ROM).

measurements. Because, for the kklc7_rec4 OP, the liquid inlet velocity is also about 0.98 m/s, $C_0 = 1.03$ is clearly a reasonable value for the void distribution parameter (Dokhane et al., 2007).

(ii) Based on the relationship between the vapour and liquid velocities used for the slip model in RAMONA, it was found that the average slip value is 1.35 for the kklc7_rec4 OP. This corresponds to a drift velocity $V_{gj} = 0.12$.

2.1. Semi-analytical bifurcation analysis

The bifurcation analysis for kklc7_rec4 OP was carried out using the bifurcation code BIFDD employing the methodology described in Dokhane (2004). As seen in Fig. 1, the kklc7_rec4 OP is located on the unstable side and lies very close to the stability boundary. The transformed stability boundary in the power–flow plane is shown along with the exclusion area in Fig. 2. Again, as in Fig. 1, the kklc7_rec4 OP is located in the unstable region. In addition, Fig. 3, which shows the nature of PAH bifurcation along the SB, indicates that for the SB branch with $N_{sub} > 2.1$, the type of PAH bifurcation is subcritical whereas, for the branch with $N_{sub} < 2.1$, supercritical PAH bifurcation is expected.

With kklc7_rec4 located in the unstable region and lying very close to the SB branch where a supercritical PAH bifurcation is expected, one can conclude that a stable limit cycle solution should be found. Moreover, a close look at the properties of the elements of the eigenvector corresponding to the eigenvalue with largest real part (responsible for the occurrence of the PAH bifurcation) reveals that only in-phase oscillations are expected to be observed when the system loses its stability.

Confirmation of these BIFDD predictions of the system behaviour for kklc7_rec4 is obtained in the following section by numerically integrating the set of 22 ODEs of this ROM for parameter values corresponding to this OP using a MATLAB code based on the Gear's algorithm.

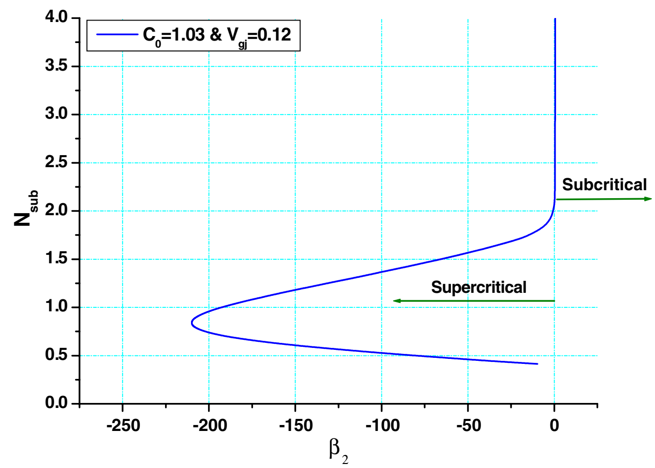

Fig. 3. Bifurcation characteristics in N_{sub}–β_2 plane for operating and design parameters corresponding to the kklc7_rec4 operational point (ROM).

2.2. Numerical simulation

Fig. 4 shows the time evolution of the amplitude of the fundamental and first mode oscillations. The development of a stable limit cycle for $n_0(t)$ is clearly seen here. This is in agreement with the bifurcation analysis prediction (supercritical PAH bifurcation), as well as with the eigenvector analysis that predicts the excitation of in-phase oscillations. The time evolution of the inlet velocities of the two channels is depicted in Fig. 5. Both channels are seen to behave in the same manner, i.e. the two inlet velocities have the same amplitude and phase. This again confirms the excitation of the in-phase oscillation mode at this OP. The oscillation frequency from the numerical results obtained using the reduced order model is 0.63 Hz.

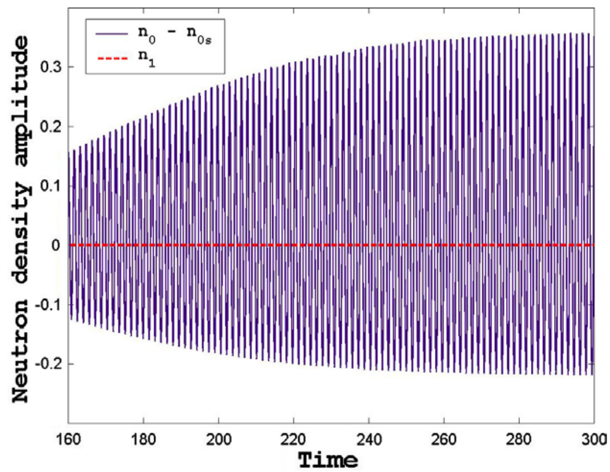

Fig. 4. Time evolution of the deviation of the fundamental mode amplitude in the ROM from the steady-state value ($n_0 - n_{0s}$) and the first mode amplitude (n_1) at the kklc7_rec4 operational point (n_{0s} is the steady-state value for $n_0(t)$).

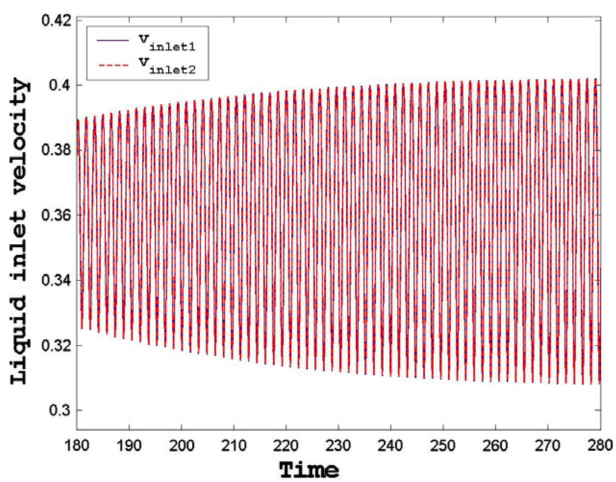

Fig. 5. Time evolution of the (liquid) inlet velocity in channel 1 (v_{inlet1}) and channel 2 (v_{inlet2}) predicted by the ROM. The two channels oscillate in-phase.

3. Stability and bifurcation analysis using RAMONA

In this section, a detailed bifurcation analysis is carried out using the system code RAMONA-5/PRESTO1.[5] Stability characteristics of kklc7_rec4 OP are determined first. Then, in order to understand the system behaviour in the neighbourhood of this OP, further analyses are performed for various nearby OPs.

3.1. Stability behaviour of the kklc7_rec4 OP using RAMONA

The stability characteristics of the kklc7_rec4 OP are investigated in this section using the system code RAMONA.

[5] This version is equivalent to RAMONA-3, but runs much faster.

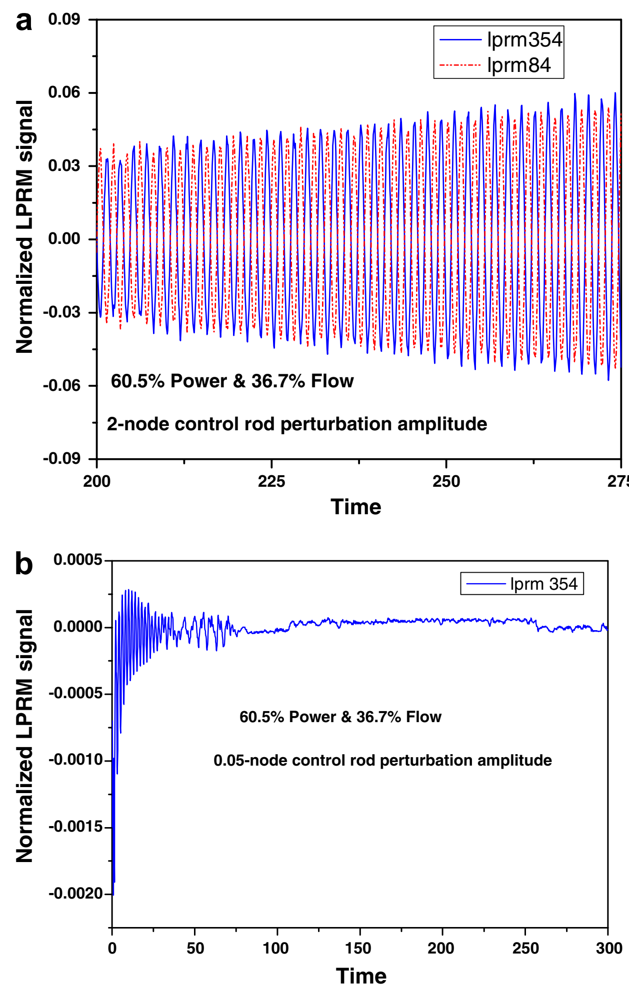

Fig. 6. Time series of the kklc7_rec4 LPRMs (RAMONA analysis) showing the occurrence of out-of-phase oscillations with a subcritical PAH bifurcation: (a) 2-node control rod perturbation amplitude and (b) 0.05-node control rod perturbation amplitude.

Fig. 6a shows the time series of LPRM84[6] and LPRM354 for a perturbation amplitude of 2 nodes.[7] A 2-node amplitude[8] control rod perturbation means that a sinusoidal (perturbation) movement of a specific control rod bank is induced with an amplitude of 2 nodes over 1 s.

Fig. 6a clearly shows the excitation of out-of-phase oscillations (with a frequency of 0.58 Hz), i.e. when the power increases in LPRM84, it decreases in LPRM354, and vice versa. In addition, the oscillation amplitudes of both LPRMs are increasing. The increasing amplitude oscillation in Fig. 6a could be indicative of: (1) a stable fixed point with an unstable limit cycle around it; (2) an

[6] LPRM84 stands for local power range monitor (LPRM) number 8 located at axial level 4. Level 4 corresponds to the highest level.

[7] This is the default value of the perturbation amplitude in RAMONA input.

[8] The Leibstadt core is modelled with 25 axial nodes in RAMONA, where each node equals 15.24 cm.

unstable fixed point with a stable limit cycle[9] around it; or (3) an unstable fixed point. A decreasing amplitude oscillation that has not been allowed to evolve long enough to determine if it would decay to zero or would saturate at some stable amplitude limit cycle, on the other hand, could be indicative of: (1) a stable fixed point with an unstable limit cycle around it; (2) an unstable fixed point with a stable limit cycle around it; or (3) a stable fixed point. Unlike the usage of ROM, presented in Section 2, that allows analytical or semi-analytical bifurcation analysis leading to determination of sub- or supercritical bifurcation along the SB, large scale system codes do not permit such determinations. Therefore, detailed numerical investigations—simulations over much longer period of time and/or simulations with different amplitude perturbations—must be carried out to determine the type of solution (bifurcation) and to uniquely identify the state of the system.

Thus, for an unstable fixed point, whatever the initially induced perturbation amplitude, oscillations always grow in amplitude. On the other hand, for the case of stable fixed point with an unstable limit cycle around it, the initial perturbation amplitude plays an important role in determining the behaviour of the system, viz. for small perturbation amplitudes the oscillations decay to the stable fixed point, whereas for a large enough perturbation amplitude the system is repelled from the unstable limit cycle orbit and the oscillations grow in amplitude. Thus, a perturbation with significantly smaller amplitude is introduced to determine the impact of perturbation amplitude on the system behaviour.

Fig. 6b shows the time series of the two LPRMs at the same OP, but with a perturbation amplitude of 0.05 node instead of 2 nodes. The figure clearly shows that the oscillation amplitude decays to a stable fixed point, indicating a clear dependence on perturbation amplitude and a dramatic change of the qualitative behaviour of the system compared with the large amplitude perturbation case. This conclusively shows that the OP is a stable fixed point with an unstable limit cycle around it. Once again, it should be borne in mind that the question of the bifurcation type leading to an observed stable limit cycle or growing/decaying amplitude oscillations in the course of BWR stability analysis using large system codes has not been raised before. It should be recalled in this context that the reduced order model had predicted a supercritical bifurcation at the kklc7_rec4 OP. This will be discussed further in a later section.

3.2. Bifurcation analysis for different Leibstadt OPs around kklc7_rec4 OP

In this section, the stability behaviour of the Leibstadt NPP is analysed in the neighbourhood of the reference operational point kklc7_rec4. A detailed local investigation is carried out to study how the solution manifold of the system varies as a function of the mass flow rate, which is here considered to be the bifurcation parameter. The results obtained are then interpreted by comparing them with the results found using our reduced order model. Note that based on the experience accumulated to date using ROMs of BWRs, only sub- and supercritical PAH bifurcations have been observed and reported during the loss of BWR stability.

It should be pointed out that the intention here has been to perform a qualitative comparison between the results found using RAMONA and those found using the PSI ROM. In other words, the objective has been to compare how the solution manifold can vary as a function of a certain bifurcation parameter, viz. the mass flow rate. In effect, the stability behaviour is investigated at the following five OPs:

- 60.5% thermal power and 36.7% mass flow rate (nominal OP, kklc7_rec4).
- 60.5% thermal power and 37.0% mass flow rate (+0.3% F OP[10]).
- 60.5% thermal power and 37.7% mass flow rate (+1% F OP).
- 60.5% thermal power and 36.4% mass flow rate (−0.3% F OP[11]).
- 60.5% thermal power and 35.7% mass flow rate (−1% F OP).

At each of these OPs, RAMONA analyses have been carried out by inducing control rod (CR) perturbations with different amplitudes, the objective being to analyse the stability behaviour for each OP and for each CR perturbation amplitude separately. Figs. 7–11 show the LPRM354 time series signals, calculated using RAMONA, for the nominal OP, +0.3% F OP, +1% F OP, −0.3% F OP and −1% F OP, respectively.

3.3. The nominal OP: 60.5% power and 36.7% mass flow

As mentioned previously, this operating point is located in the exclusion area in the power–flow map of the Leibstadt NPP. For a small amplitude perturbation (0.05-node control rod perturbation), the power decays to the stable steady-state solution as shown in Fig. 7a, while a perturbation amplitude of 0.1-node of the control rod leads to growing oscillation amplitudes (Fig. 7b). Increasing the initial perturbation further to 2 nodes also results in growing amplitude oscillations (Fig. 7c). Phenomenologically, this indicates that beside the stable fixed point solution, an unstable limit cycle solution exists around this operational point.

[9] Historically, this type of solution was ascribed to the system at this OP, with the argument that, at longer times, the growing oscillation amplitude would saturate to a stable periodic solution (stable limit cycle).

[10] Means that the mass flow for this OP is higher than that for the nominal OP by 0.3%.

[11] That the mass flow for this OP is less than that for the nominal OP by 0.3%.

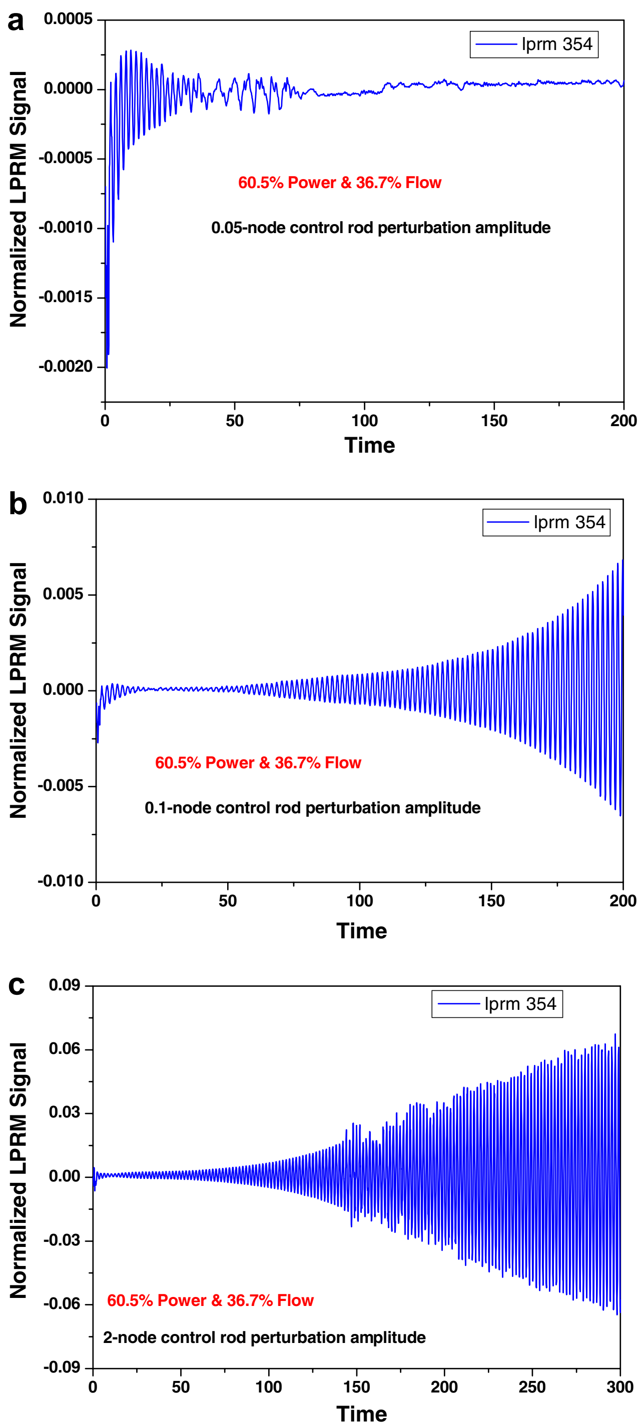

Fig. 7. Nominal OP time series of Leibstadt LPRM showing the occurrence of a subcritical PAH bifurcation: (a) 0.05-node control rod perturbation amplitude, (b) 0.1-node control rod perturbation amplitude and (c) 2-node control rod perturbation amplitude.

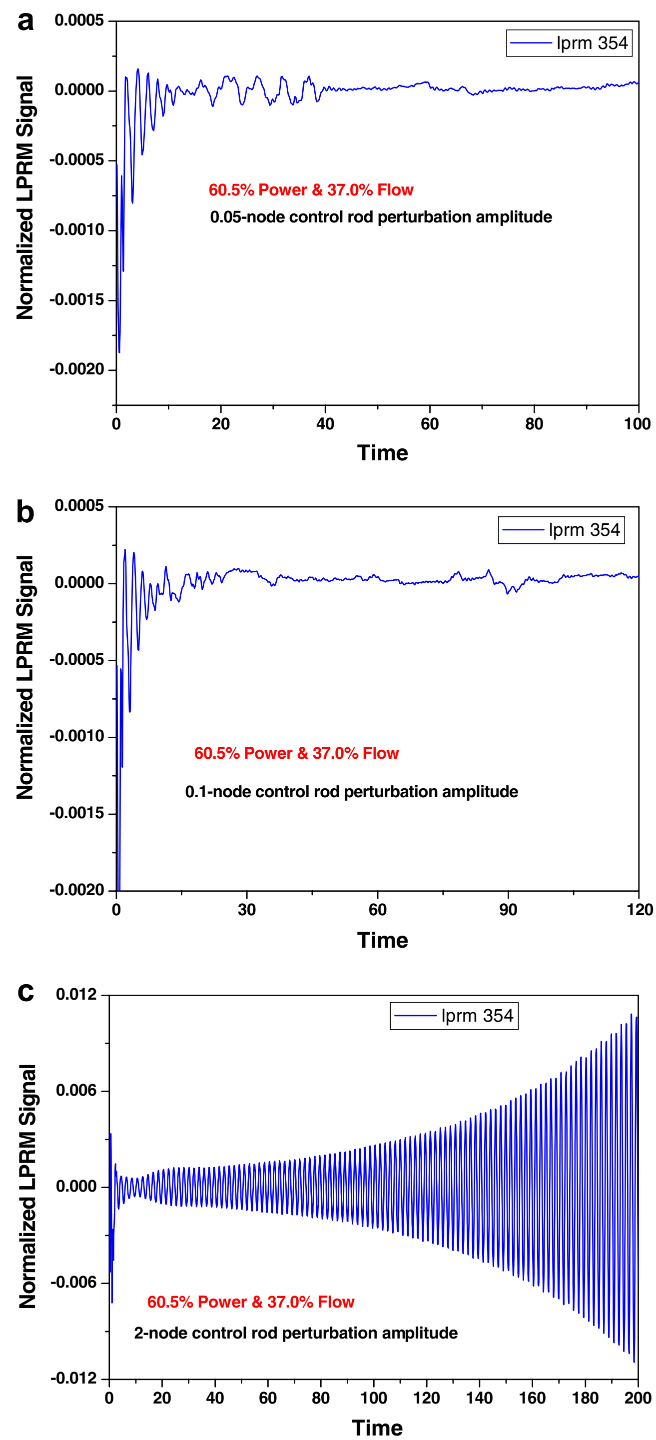

Fig. 8. +0.3% F OP time series of Leibstadt LPRM showing the occurrence of a subcritical PAH bifurcation: (a) 0.05-node control rod perturbation amplitude, (b) 0.1-node control rod perturbation amplitude and (c) 2-node control rod perturbation amplitude.

3.4. The +0.3% F OP: 60.5% power and 37.0% mass flow

Fig. 8 shows clearly that, at this operational point, the system again has two different behaviours depending on the perturbation amplitude. Thus, for 0.05-node and 0.1-node control rod perturbation amplitudes, the power oscillations are seen to decay to the stable steady-state solution (stable fixed point) as shown in Fig. 8a and b, while, for a 2-node control rod perturbation amplitude, oscillations with growing amplitudes are observed. Therefore, an unstable limit cycle solution also exists

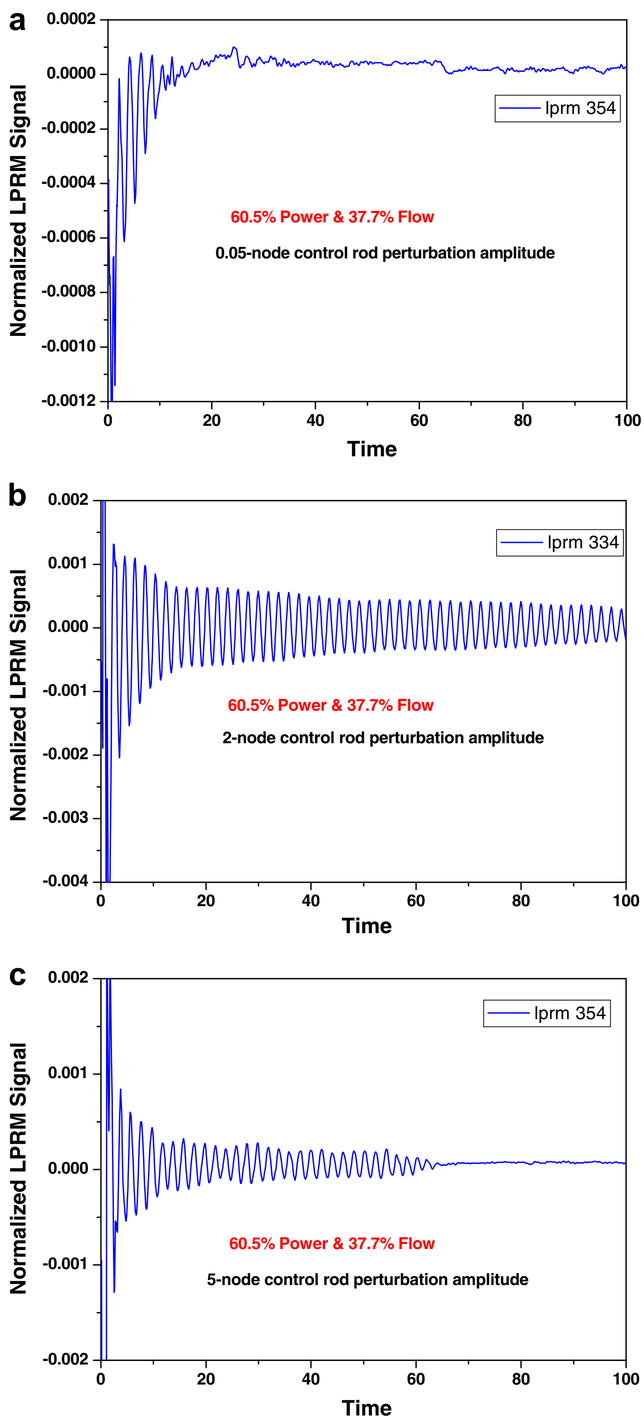

Fig. 9. +1% F OP time series of Leibstadt LPRM showing a stable fixed point solution: (a) 0.05-node control rod perturbation amplitude, (b) 2-node control rod perturbation amplitude and (c) 5-node control rod perturbation amplitude.

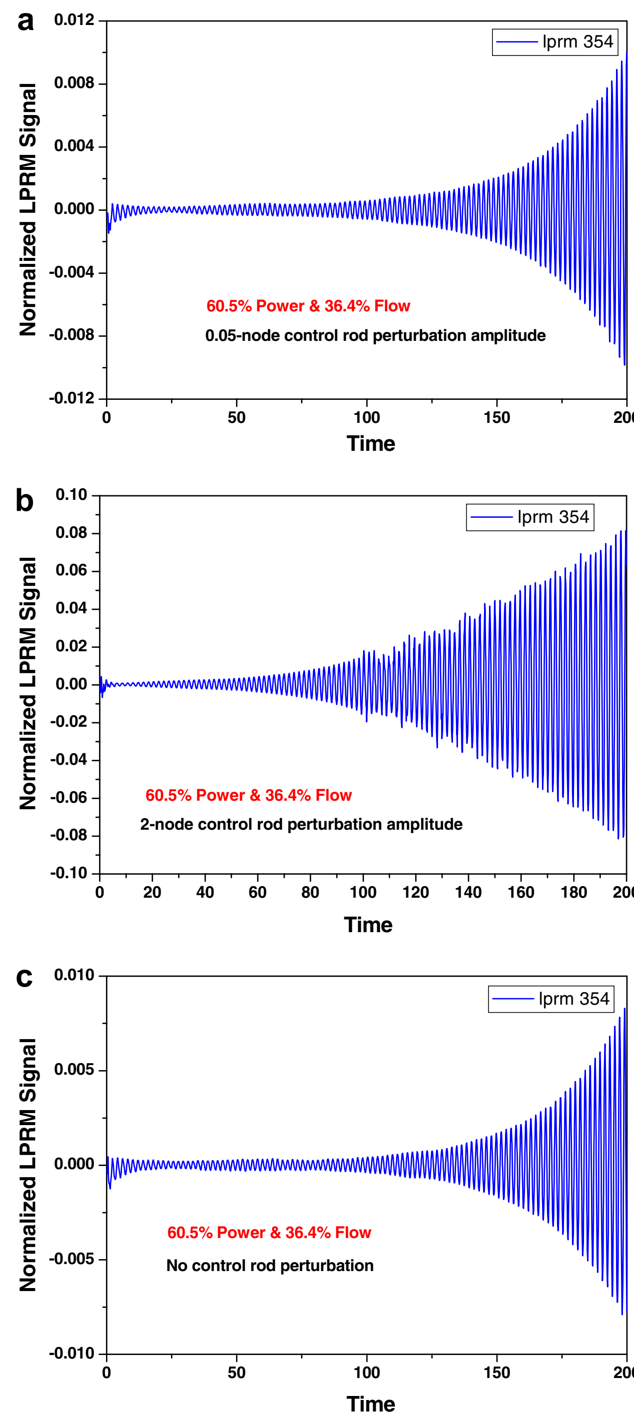

Fig. 10. −0.3% F OP time series of Leibstadt LPRM showing an unstable fixed point solution: (a) 0.05-node control rod perturbation amplitude, (b) 2-node control rod perturbation amplitude and (c) no control rod perturbation.

around this OP. Moreover, while at the nominal OP a 0.1-node perturbation is enough to kick the system out of the stable fixed point's basin of attraction (growing oscillations (Fig. 7b)), the same perturbation at +0.3% F OP is not enough to destabilize the system (decaying oscillations (Fig. 8b)). This can be explained by the difference in the amplitude of the unstable limit cycles at the two different OPs (nominal and +0.3% F). Consequently, for a perturbation amplitude of 0.1 node or less, the system in phase space remains in the basin of attraction of the stable fixed point and is hence attracted by it (Fig. 8a and b).

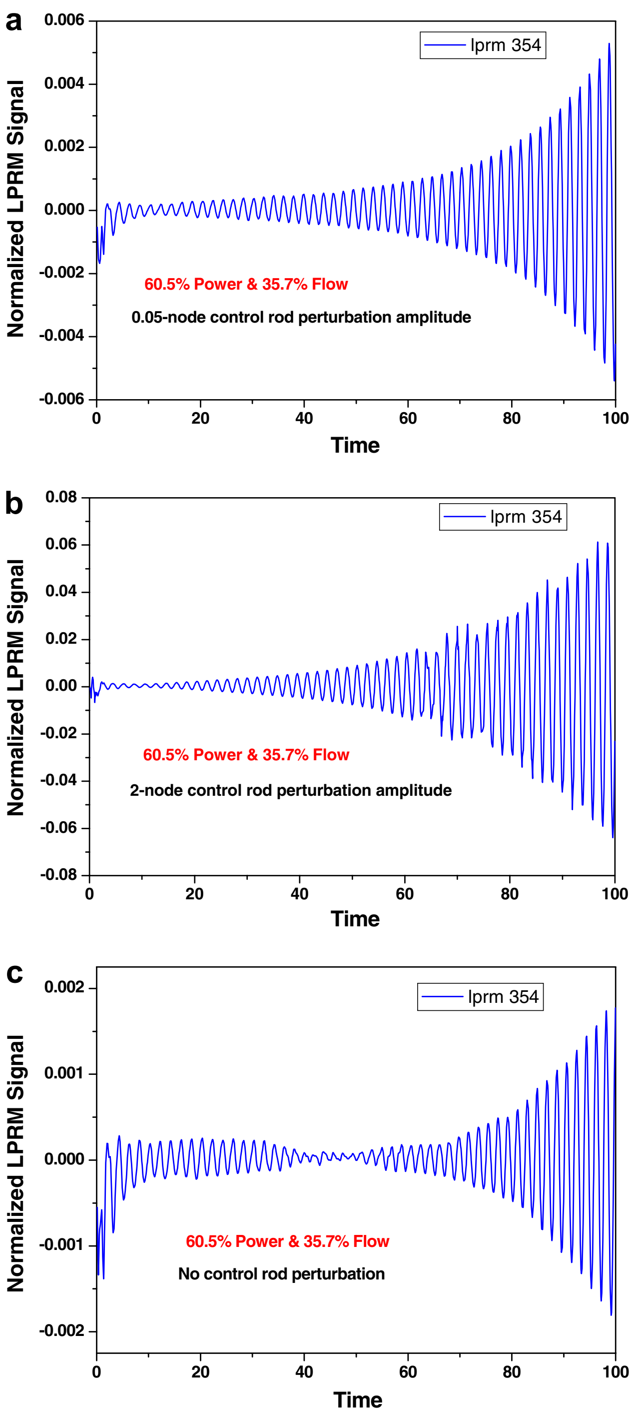

Fig. 11. −1% F OP time series of Leibstadt LPRM showing an unstable fixed point solution: (a) 0.05-node control rod perturbation amplitude, (b) 2-node control rod perturbation amplitude and (c) no control rod perturbation is induced.

3.5. The +1% F OP: 60.5% power and 37.7% mass flow

The behaviour of the reactor at this operational point is shown in Fig. 9. From parts (a) and (b) of this figure, it is clearly seen that the system is stable (stable fixed point solution) for both 0.05- and 2-node control rod perturbation amplitudes. In order to rule out the existence of a large

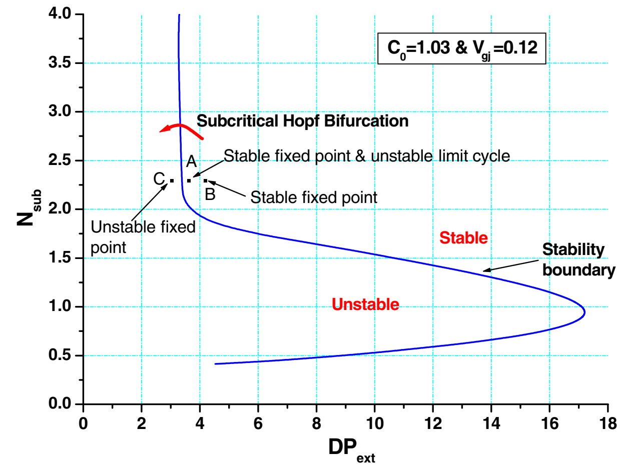

Fig. 12. Different types of solutions during a subcritical PAH bifurcation occurrence, as predicted using the PSI reduced order model.

amplitude unstable limit cycle solution,[12] the control rod perturbation amplitude was increased to 5 nodes.[13] Fig. 9c clearly shows that the analysed OP is indeed a stable fixed point.

3.6. The −0.3% F OP: 60.5% power and 36.4% mass flow

For the −0.3% F OP, Fig. 10a and b shows that the oscillation amplitude grows independently of the perturbation amplitude (0.05- or 2-node control rod perturbation), i.e. the system is unstable (unstable fixed point solution). To rule out the possibility of an unstable limit cycle with very small amplitude, a case has been analysed in which there was no induced control rod perturbation at all, i.e. only the numerical noise, assumed to be very small, acts as a perturbation. Results shown in Fig. 10c clearly confirm that the system is indeed unstable, i.e. an unstable fixed point is indeed the solution at this OP.

3.7. The −1% F OP: 60.5% power and 35.7% mass flow

The behaviour of the system at the −1% F OP is the same as that for the −0.3% F OP (see Fig. 11a–c), i.e. once again the solution is seen to be an unstable fixed point.

3.8. Interpretation and discussion

At first glance, it may seem quite peculiar that the qualitative behaviour (solution type) of the system changes dramatically within a small range of the mass flow rate (from 37.7% to 35.7%), i.e. from a stable fixed point solution at the +1% F OP, to stable fixed points with unstable limit cycle solutions at the +0.3% F and nominal OPs, and then to unstable fixed point solutions at the −0.3% and −1% F

[12] If a large amplitude limit cycle exists, larger perturbation amplitudes are needed to take the system outside the limit cycle.

[13] A control rod perturbation amplitude of 5 nodes is considered to be a very large perturbation.

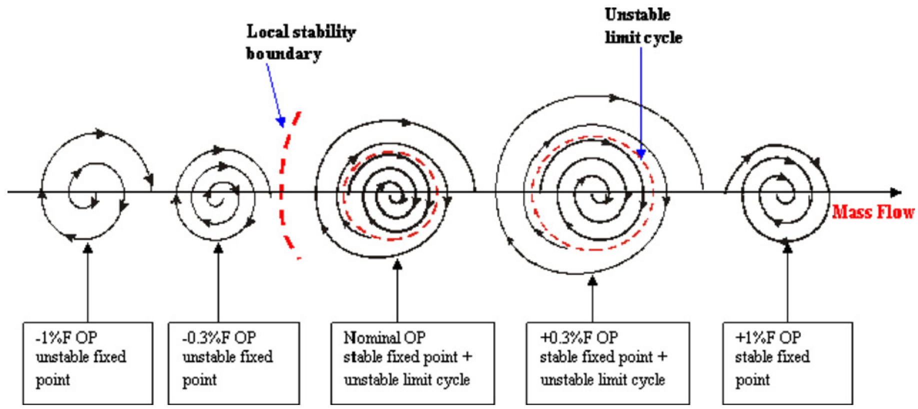

Fig. 13. Scheme showing the different solution types encountered when the mass flow is varied for the nominal Leibstadt OP.

OPs. This is, however, quite consistent with the predictions of the PAH bifurcation theorem and clearly indicates the behaviour of a dynamical system that goes through a subcritical PAH bifurcation. Moreover, while carrying out semi-analytical bifurcation analysis using our reduced order model, such system behaviour has been observed. Fig. 12 shows the different solutions that exist close to the SB when such a bifurcation is expected for the reduced order model. Note that region in N_{sub}–DP_{ext} space where the ROM predicts subcritical PAH bifurcation is different from the region around the nominal kklc7_rec4 OP. Hence, the similarity in the variation of the solution type between the BWR system analysis with RAMONA and the reduced order model analysis is only qualitative.

Thus, points A–C in Fig. 12 are located in a region where a subcritical PAH bifurcation occurs.[14] Because the operational point A is in the stable region and close to the stability boundary, both a stable fixed point and an unstable limit cycle solution are found. Point B is inside the stable region but far from the SB. Here, only a stable fixed point solution is observed. Finally, point C is in the unstable region and the unstable fixed point is, therefore, the only solution of the system. Again, it needs to be emphasized that such a change of solution type happens only because of the occurrence of a subcritical PAH bifurcation. Therefore, based on the reduced order model findings, two important conclusions can be drawn:

1. The change of the solution type from +1% F OP to −1% F OP in the RAMONA calculation can be explained only by the occurrence of a subcritical PAH bifurcation during the loss of system stability.
2. The nominal, +0.3% F, and +1% F OPs are located in the stable region, while the −0.3% F and −1% F OPs are located in the unstable region. Consequently, a local stability boundary exists between the nominal OP and the −0.3% F OP. The schema shown in Fig. 13 summarizes the results found in this study.

4. The correspondence hypothesis: stable (unstable) limit cycle vs. supercritical (subcritical) PAH bifurcation

For a PAH bifurcation to occur, three conditions imposed by the Hopf bifurcation theorem have to be fulfilled (Nayfeh and Balachandran, 1995). These conditions can easily be verified when using models represented by a system of ODEs, as is the case with reduced order models, since analytical bifurcation analysis can then be carried out using a bifurcation code like BIFDD. However, for models based on PDEs, as those used by the system code RAMONA, one does not presently have the capability to check the fulfilment of the conditions for the occurrence of a PAH bifurcation. Nevertheless, certain guidelines may be suggested by considering the following facts:

1. A supercritical PAH bifurcation is characterized by the appearance of stable limit cycle solutions inside the linear unstable region close to the SB, while a subcritical PAH bifurcation is characterized by a stable fixed point and an unstable limit cycle solution inside the linear stable region close to the SB.
2. Only sub- or supercritical PAH bifurcations have been observed and reported so far during the loss of system stability in the context of BWR stability analysis using reduced order models. Therefore, one can confidently assume that these two types of bifurcation are the only ones that can be expected to occur when a BWR loses its stability.
3. During the occurrence of a subcritical PAH bifurcation, a turning point may exist in which large amplitude stable limit cycles[15] are observed in the linear unstable region.

Therefore, excluding the large amplitude stable limit cycle case, i.e. turning point, the following "correspondence hypothesis" is suggested in the framework of BWR stability analysis using system codes:

[14] From analysis using our reduced order model in conjunction with the bifurcation code BIFDD.

[15] In Rizwan-uddin (2000), the amplitude of the stable limit cycle due to the turning point was found to exceed 300%.

When a BWR system loses its stability, the observation of a stable limit cycle is indication of the occurrence of a supercritical PAH bifurcation, while the existence of an unstable limit cycle indicates the occurrence of a subcritical PAH bifurcation.

5. Reduced order model assessment against the system code RAMONA

Comparing the kklc7_rec4 OP predictions obtained using RAMONA (Section 3.1) and the PSI reduced order model (Section 2.2), one can draw several conclusions. Thus, firstly, the reduced order model is seen to yield a good prediction of the location of the OP with respect to the stability boundary (both tools predict the OP to be very close to SB), as well as of the oscillation frequency value. However, the discrepancy is obvious between the model results and those of RAMONA concerning the excited oscillation mode (in-phase or out-of-phase) and the nature of PAH bifurcation. This is not surprising, keeping in mind that: (a) the reduced order model is highly simplified with the entire BWR core lumped into two representative channels; and (b) the design and operating parameters used in the ROM are core-average values.

It should be stressed, however, that the main objective of ROM studies is to permit analysis of the complete solution manifold of the BWR system. Thus, as demonstrated by Dokhane et al. (2007), the application of a suitable ROM can provide deep generic insights into the complex processes determining BWR stability. As discussed in this previous paper, the PSI ROM was developed while trying to respect two conflicting requirements, viz. faithful representation of the physical model (closeness to the system code RAMONA) and simplicity. Clearly, the latter feature can result in certain discrepancies, and further improvements in the ROM modeling are called for if the differences with respect to the system code predictions are considered important in a given context.

The PSI reduced order model predicted in-phase oscillations, while out-of-phase oscillations are predicted by RAMONA for the reference Leibstadt OP. This discrepancy is mainly because of the limitations of the feedback reactivity model for the mode coupling ($\rho_{10}(t)$) and ($\rho_{01}(t)$) (Dokhane et al., 2007). Accordingly, it is suggested that an improved model for the feedback reactivities in mode coupling be derived for future work.

The discrepancy in predicting the nature of PAH bifurcation is due mainly to the uncertainties in evaluating the design and operating parameters as core-average values. In Dokhane (2004), it was found that changing the value of certain parameters, e.g. the drift flux model parameters (C_0 and V_{gj}) or the inlet pressure loss coefficient (K_{inlet}), can change the nature of PAH bifurcation. In other words, it was observed that a SB branch that was associated with subcritical PAH bifurcation can become supercritical, or vice versa. This means that a small discrepancy in evaluating one or more of such parameters may lead to a wrong prediction as regards the nature of the PAH bifurcation. Also the recent ROM studies of Dokhane et al. (2002) and Zhou and Rizwan-uddin (2002) have shown that the type of PAH bifurcation encountered can be very sensitive to the modelling assumptions made. Clearly, since the PSI ROM has various modelling assumptions which differ from those of RAMONA, it is not surprising to observe the discrepancy in PAH bifurcation type as predicted by the two models.

Despite the quantitative limitations of the PSI ROM, it has been clearly demonstrated in Section 3 that ROM predictions can be invaluable for the proper interpretation of the complicated results which are sometimes obtained in BWR stability analysis using large system codes.

6. Summary and conclusions

The PSI BWR reduced order model has been used to analyse a specific operational point of the Leibstadt NPP (kklc7_rec4). The results obtained have been compared to those of the system code RAMONA for the same OP, thus permitting a direct assessment of the performance of the PSI ROM in terms of both its applicability and its limitations.

It has been seen that, for the case considered, the reduced order model very well predicts the frequency of the oscillations and also localizes the analysed OP in an appropriate region close to the stability boundary. However, clear discrepancies have been found between the ROM model results and those of RAMONA as regards the prediction of the oscillation mode (in-phase and out-of-phase) and the nature of PAH bifurcation at this operational point. The inability of the ROM to predict the out-of-phase oscillation mode at this OP is due to the limitations of the feedback reactivity model for the mode coupling, while the discrepancy in predicting the nature of PAH bifurcation is mainly due to the uncertainties in evaluating the design and operating parameters adequately. These "negative" results concerning the reduced order model's quantitative performance are in fact not surprising since the model, in a relative sense, is highly simplified, with the entire BWR core being lumped into just two representative channels, and the design and operating parameters calculated for a specific OP being simply core-average values.

A detailed numerical bifurcation analysis was carried out using the system code RAMONA in the immediate neighbourhood of the reference OP. The main conclusion to be drawn from this investigation is that, although the reduced order model has limitations in quantitative performance, its results are of paramount importance in analysing and interpreting the results obtained with RAMONA. It is a consequence of the "bridge" built between the ROM and RAMONA that a subcritical

PAH bifurcation has been identified, for the first time, in the course of BWR stability analysis using a system code.

As a general conclusion, it should be stressed that detailed quantitative studies for specific NPP operational points are still not possible using a reduced order model, and this remains a challenge. Thus, reduced order models, with respect to detailed system codes, still need to be considered as valuable *complementary* tools, and *not* as alternatives.

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