Pergamon

Ann. Nucl. Energy, Vol. 21, No. 5, pp. 321–324, 1994

0306–4549/94 $7.00+0.00

TECHNICAL NOTE

STEADY-STATE CHARACTERISTICS BASED MODEL FOR CENTRIFUGAL PUMP TRANSIENT ANALYSIS

RIZWAN-UDDIN

Department of Mechanical, Aerospace and Nuclear Engineering, University of Virginia, Charlottesville, VA 22903-2442, U.S.A.

Abstract—A simple phenomenological model to simulate fast transients expected in centrifugal pumps in nuclear power plants, is developed. The model, which is based on steady-state characteristics of the pump, has been used successfully to simulate extremely fast startup and stopping of centrifugal pumps relevant to nuclear reactor safety studies. Comparison with experimental data shows very good agreement.

INTRODUCTION

The need for a simple and efficient model for fast pump transient analysis has not diminished over the years. The difficulty in such modeling arises from the complicated nature of fluid flow inside the pump under time-varying conditions. Even models based on first principles often give poor quantitative agreement with experimental data. This is often due to unavoidable approximations made during the course of mathematical development or in the numerical solution procedure. These approximations are sometimes so severe that the mathematical models developed for transient analyses show poor quantitative agreement even for time-independent problems (Tsukamoto *et al.*, 1986; Tsukamoto and Ohashi, 1982).

The current trend to alleviate the problem is to include more details of the fluid flow inside the pump. Details of steady and unsteady flows inside the casing have been a topic of research, and it has been shown that the unsteady torque for transient or sinusoidal flowrate or angular velocity fluctuation can be divided into three components: quasi-steady, apparent mass and wake (Saito, 1982). The effect of mass of water in the pipeline, valve opening and starting time on the transient characteristics of a pump were studied experimentally and on the basis of 1-D equations of motion using an analogue computer (Saito, 1982). Also, importance of cavitation in pumps makes it necessary to take into account the effects of various parameters on cavitation and hence, on the steady-state and transient behavior of the pump. Although, including cavitation and other effects in the dynamics of centrifugal pumps makes the analysis much more complicated, such detailed transient studies, if correctly carried out, can be used to develop more accurate centrifugal pump models.

Including more details in models so far has made the new models for fast transient analysis more complicated without much improving their accuracy. Even if the accuracy of these models can be improved by adding even more details, they will be too complicated to be used in computer codes developed to analyze large systems—such as a primary loop of a nuclear reactor—in which pump dynamics calculation represents only a small fraction of the total computational effort. Although mathematical models based on first principles, which take into account the details of the pumping action, are desirable for the development of new and improved pumps, empirical relations currently being used (often supplied by the manufacturer)—which relate the flowrate to the pump head for different shaft speeds—are sufficiently accurate to analyze steady-state and *slow* transient operations of centrifugal pumps. Due to the relative accuracy and ease of computation of these experimentally obtained steady-state characteristic equations, it is desirable to utilize them to develop phenomenological models to determine pump dynamic behavior under *rapidly changing conditions*. One obvious advantage in this approach is that the dynamical model can be developed such that in time-independent cases it exactly satisfies the steady-state characteristic equation and hence, the initial and final states of the pump at the beginning and end of the transient will necessarily match the known steady-state characteristics.

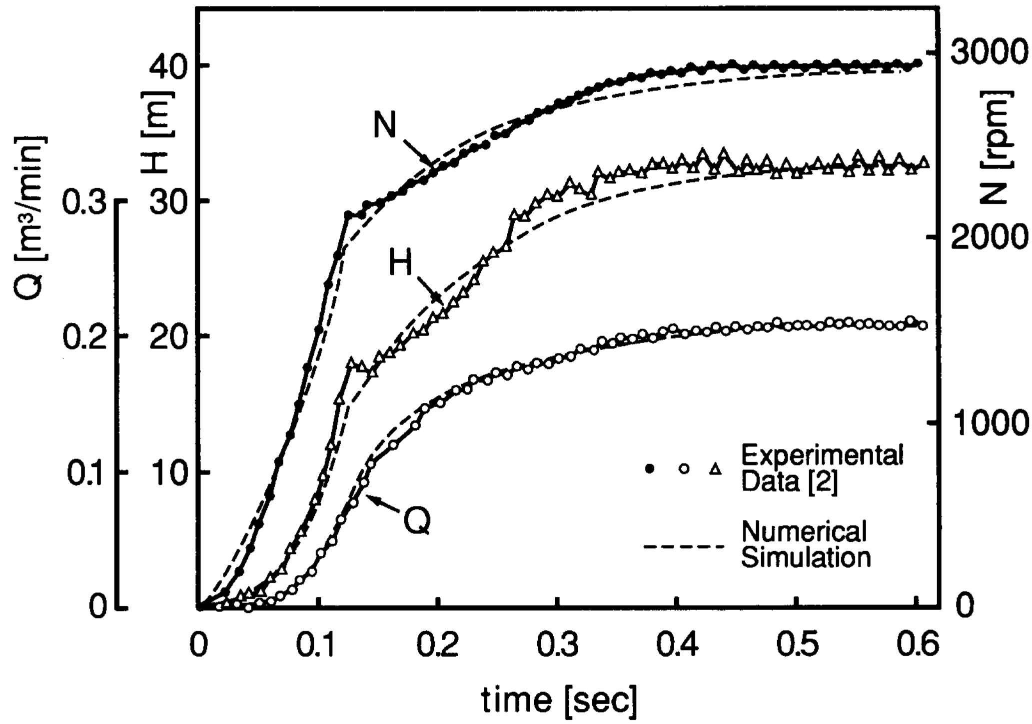

Fig. 1. Comparison of numerical values with experimental data. Shown are the numerical values for shaft speed N, pump head H and flowrate Q as a function of time along with the experimental data for a fast pump startup. Experimental data is from Tsukamoto and Ohashi (1982).

MODEL

In this paper we present a simple phenomenological model, obtained using the steady-state characteristic equation, to simulate fast centrifugal pump transients. The model relates the pump head $H(t)$ [$H \equiv P/(\rho g)$] and flowrate $Q(t)$ with shaft speed $N(t)$. (A separate equation based on motor dynamics can be added to obtain shaft speed as a function of time.) The model, besides using the steady-state characteristic equation for centrifugal pumps, is based on the hypothesis that rate of change of head $H(t)$, is proportional to the rate of change of the square of the shaft speed $N(t)$:

$$\frac{\mathrm{d}H(t)}{\mathrm{d}t} = aN(t)\frac{\mathrm{d}N(t)}{\mathrm{d}t}, \tag{1}$$

$$b\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = -\{cH(t) + \mathrm{d}Q^2(t) + eN^2(t)\}, \tag{2}$$

where a is chosen such that $H(t)$ as $t \to \infty$ is equal to the head that corresponds to the shaft speed $N_\infty \equiv N(t \to \infty)$ (this is equivalent to specifying the extent of the valve opening at the exit); b is equal to $2l_{eq}/A_0$, where the pump is of equivalent length l_{eq} and of constant area A_0; and c, d and e are the coefficients of the H, Q^2 and N^2 terms in the steady-state characteristic equation, respectively. Note that this model, in the absence of any change in shaft speed N, satisfies the steady-state characteristic equation exactly. Thus, for a given pump, steady-state characteristics will provide the constants c, d and e, and b can be approximated from pump geometry.

In order to solve the set of equations, equation (1) can first be integrated exactly to obtain $H(t)$:

$$H(t) - H(0) = \frac{a}{2}[N^2(t) - N^2(0)],$$

which can then be used to determine a such that $H(t \to \infty)$ corresponds to shaft speed $N(t \to \infty)$. For given shaft speed $N(t)$ and pump head $H(t)$, equation (2), which is a form of Riccatti equation, can then be solved numerically.

RESULTS

To test the accuracy of the model various transients have been simulated. Figure 1 shows the experimental data from Tsukamoto and Ohashi (1982) of a fast

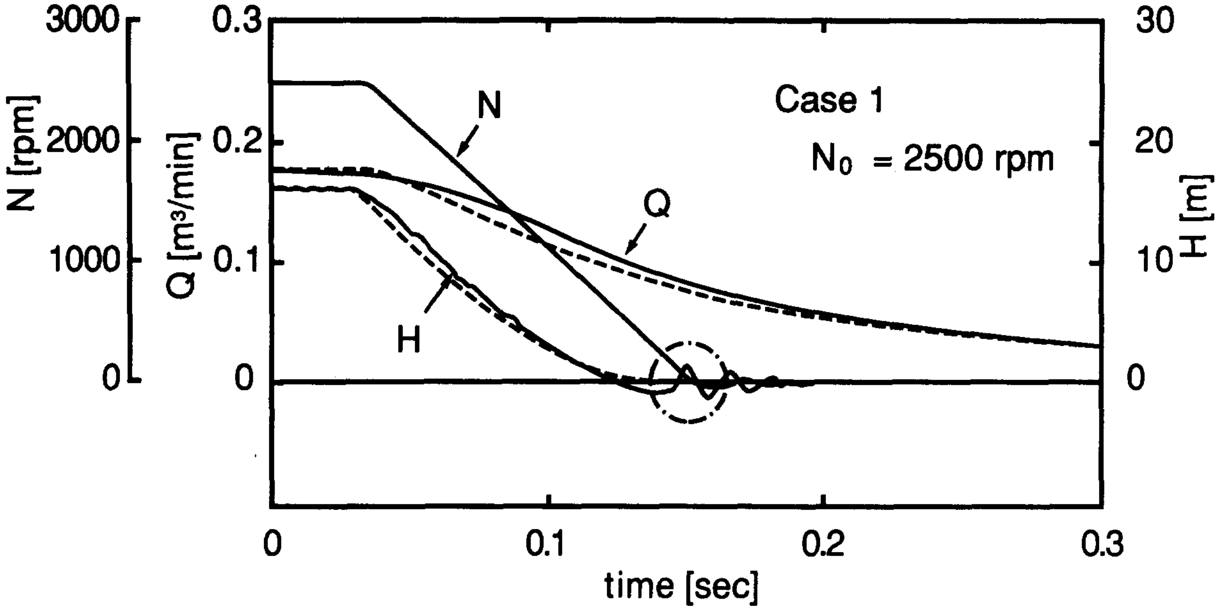

Fig. 2. Comparison of numerical simulation results with experimental data for an extremely fast pump stopping action. Shown are the experimental (———) and simulation values (----) for shaft speed N, pump head H and flowrate Q. Experimental data is from Tsukamoto *et al.* (1986)

startup transient. Rapid acceleration in shaft speed in this particular experiment was obtained by activating an electromagnetic clutch which connected the stationary pump shaft with the idling motor. To simulate the experiment, the experimental shaft speed $N(t)$ shown in Fig. 1 was approximated by two exponentials:

$$N(t) = \begin{cases} A(e^{Bt}-1), & t \leqslant 0.12\ \mathrm{s}, \\ C(1-e^{-Dt}), & t \geqslant 0.12\ \mathrm{s}, \end{cases}$$

where A, B, C and D are 21.0, 7.80, 49.2 and 8.96, respectively. Equations (1) and (2) were then solved for pump head $H(t)$ and flowrate $Q(t)$. Results of the numerical analysis are also shown in Fig. 1. The simulation, for $t \geqslant 0.12$ s, approximates the shaft speed by an exponential function whose second derivative is negative, whereas the experimental data for shaft speed, for $0.12\ \mathrm{s} \leqslant t < 0.25$ s, shows a positive second derivative. Despite this shortcoming in the approximation of shaft speed, the fast startup transient is very well represented by the simple model given in equations (1) and (2) and it is clear that the agreement between the experimental data and numerical simulation would increase even further if a better approximation to the shaft speed $N(t)$ was used in the simulation.

Transient characteristics of a centrifugal pump during the stopping period was also simulated and compared with the experimental data reported in Tsukamoto *et al.* (1986). Total pressure rise, instantaneous rotational speed and flowrate in these experiments were measured for different stopping schemes in tests with a rapid deceleration rate. An electromagnetic brake, which connected the rotating pump shaft with the stationary brake disk after disconnecting the shaft from the rotating motor, was used to achieve the extremely high deceleration rate in rotational speed. Other methods were used to achieve slower deceleration rates Tsukamoto *et al.* (1986). Figure 2 shows the experimental data for Case 1 in Tsukamoto *et al.* (1986), which had the fastest deceleration rate reported. To simulate the experiment, the experimental shaft speed $N(t)$ shown in Fig. 2 was approximated by three straight lines:

$$N(t) = \begin{cases} N_0 & t \leqslant 0.03\ \mathrm{s} \\ N_0\left[1-\dfrac{(t-0.03)}{T}\right] & 0.03 \leqslant t \leqslant 0.15\ \mathrm{s} \\ 0 & t \geqslant 0.15\ \mathrm{s} \end{cases}$$

where N_0 and T are 2500 rev min^{-1} and 0.12 s, respectively. For the rotational speed given above, equations (1) and (2) were solved for pump head and flowrate.

Results of this simple simulation are also shown in Fig. 2. Once again the transient during the fast stopping periods is extremely well represented by the simple model. Note the experimental observation that the flowrate does not become zero immediately after the rotational speed reaches zero, is also well captured by the model.

SUMMARY

There has been a need to develop mathematical pump models that can be used efficiently in large computer codes to simulate unsteady pump operation, especially during fast startup and stopping periods as encountered in nuclear reactor safety studies. In this paper we have presented a dynamical model for centrifugal pumps that can be used in nuclear reactor safety analyses. This model is based on phenomenological arguments and satisfies the steady-state characteristic equation exactly. Comparison with experimental data for starting and stopping actions has shown excellent agreement.

REFERENCES

Saito S. (1982) The transient characteristics of a pump during start up. *Bull. JSME* **25**, 372–379.

Tsukamoto H. and Ohashi H. (1982) Transient characteristics of a centrifugal pump during starting period. *J. Fluids Engng* **104**, 6–14.

Tsukamoto H., Matsunaga S., Yoneda H. and Hata S. (1986) Transient characteristics of a centrifugal pump during stopping period. *J. Fluids Engng* **108**, 392–399.